

Identifying Stars of Mass $> 150M_{\odot}$ from Their Eclipse by a Binary Companion

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ABSTRACT

We examine the possibility that very massive stars greatly exceeding the commonly adopted stellar mass limit of $150M_{\odot}$ may be present in young star clusters in the local universe. We identify ten candidate clusters, some of which may host stars with masses up to $600M_{\odot}$ formed via runaway collisions. We estimate the probabilities of these very massive stars being in eclipsing binaries to be $\gtrsim 30\%$. Although most of these systems cannot be resolved at present, their transits can be detected at distances of 3 Mpc even under the contamination of the background cluster light, due to the large associated luminosities $\sim 10^7 L_{\odot}$ and mean transit depths of $\sim 10^6 L_{\odot}$. Discovery of very massive eclipsing binaries would flag possible progenitors of pair-instability supernovae and intermediate-mass black holes.

Key words: binaries: general – galaxies: star clusters

1 INTRODUCTION

Many observations support the statistical argument that the upper limit to initial stellar masses is $\sim 150M_{\odot}$ for Pop II/I stars (Figer 2005; Zinnecker & Yorke 2007). However, this common notion is challenged by the recent spectroscopic analyses of Crowther et al. (2010), in which star clusters NGC 3603 and R136 are found to host several stars with initial masses above this limit, including one star R136a1 with a current mass of $\sim 265M_{\odot}$. Also, candidate pair-instability supernovae, which require progenitors with masses above $200M_{\odot}$, have been observed in the low redshift universe (Gal-Yam et al. 2009). Therefore, it is worth exploring methods to confirm the existence of a very massive star (VMS), defined here as a star with a stellar mass significantly greater than the stellar mass limit, i.e. $M \gtrsim 200M_{\odot}$.

Unless the VMS is very close by, it is extremely difficult to spatially resolve the VMS from stars in its vicinity. Indeed, the central component of R136 was once thought to be an extremely massive $\gtrsim 10^3M_{\odot}$ star (Cassinelli, Mathis & Savage 1981), before Weigelt & Baier (1985) resolved it as a dense star cluster via speckle interferometry. As for spectroscopic measurements, verification of a single VMS is further complicated by the fact that the effective temperature T_{eff} of Pop I stars above 10^2M_{\odot} depends very weakly on mass, with $\log(T_{eff}/K) \approx 4.7\text{--}4.8$ (Bromm, Kudritzki & Loeb 2001) for stars between $10^2\text{--}10^3M_{\odot}$. Moreover, a hot evolved star with an initial mass below 10^2M_{\odot} can nevertheless reach these temperatures in its post main-sequence evolution and mimic a VMS.

The most accurate method of constraining the stellar masses of distant stars is by measuring the radial velocity and light curves of the star in an eclipsing binary (Bonanos 2009; Torres, Andersen & Giménez 2010). The light curve provides a wealth of information about the binary, including its orbital period, inclination, eccentricity, as well as the fractional radii and flux ratio of the binary members. The radial velocities found from a double-lined spectroscopic binary further provide the mass ratio of the binary. With the above information, the individual masses of each star in the binary can be calculated via Kepler’s third law. Searches for massive eclipsing binaries in star clusters within our own Galaxy are already underway (Koumpia & Bonanos 2011), and techniques have been suggested for binary searches in other galaxies (Bonanos 2012).

In this *Letter*, we estimate the masses and properties of VMSs that may have formed via collision runaways in a number of very young, dense, and massive star clusters in the local universe. We calculate the probability of these VMSs to be in eclipsing binaries, and find their expected transit depths and observability.

2 VERY MASSIVE STARS

Shortly after a dense star cluster forms, its most massive constituents sink to the center via dynamic friction and form a central subsystem of massive stars. In sufficiently dense environments, these massive stars may undergo runaway collisions and merge into a single VMS

(Gürkan, Freitag & Rasio 2004; Freitag, Gürkan & Rasio 2006), possibly up to $\sim 10^3 M_\odot$. Portegies Zwart et al. (2006) gives a fitting formula for the stellar mass m_r of the final runaway product, calibrated by N-body simulations for Salpeter-like mass functions:

$$m_r \sim 0.01 M_C \left(1 + \frac{t_{rh}}{100 \text{ Myr}}\right)^{-\frac{1}{2}}, \quad (1)$$

where t_{rh} is the relaxation time,

$$t_{rh} \approx 200 \text{ Myr} \left(\frac{r_{vir}}{1 \text{ pc}}\right)^{\frac{3}{2}} \left(\frac{M_C}{10^6 M_\odot}\right)^{\frac{1}{2}} \frac{\langle m \rangle}{M_\odot}. \quad (2)$$

Here M_C is the cluster mass, r_{vir} is its virial radius, and $\langle m \rangle \approx 0.5 M_\odot$ is the average stellar mass.

Using the compilation of stars clusters in the local universe and their properties from Portegies Zwart, McMillan & Gieles (2010), we have listed in Table 1 several young, dense star clusters that may host a runaway collision product of mass $\gtrsim 200 M_\odot$ which may have not yet ended its life as a star. We restrict our sample to clusters with mean determined ages younger than 3.5 Myr. This may already be insufficiently selective, as stars born with masses $\gtrsim 200 M_\odot$ are expected to have lifetimes of only 2-3 Myr (Yungelson et al. 2008); however, in the runaway collision scenario, the VMS builds up its extraordinary mass via mergers over $\sim 1 - 2$ Myr, and therefore its host cluster may have an age exceeding the 2-3 Myr limit. Of course, these observed cluster properties should not be taken as certain; for example, Úbeda, Maíz-Apellániz & MacKenty (2007) find the ages of NGC 4214 I-A and I-B to be $\sim 4-5$ Myr, likely too old for a VMS to be present. Conversely, there may be candidate clusters with VMSs that we have missed. The predicted runaway masses are only approximate, but give a sense of the mass range of VMSs that may lurk at the center of these very young and dense clusters.

Alternatively, if feedback effects are moderate, it may be possible for a protostar to grow without a fixed mass limit via mergers or via the accretion of extremely dense gas. In this case, the mass of the most massive star m_u formed in a molecular cloud scales with the mass of that cloud, and thus will be correlated with the mass of its eventual host cluster (Larson 1982, 2003; Weidner, Kroupa & Bonnell 2010):

$$m_u \approx 1.2 M_C^{0.45}. \quad (3)$$

If the above relationship is valid for cluster masses $> 5 \times 10^4 M_\odot$, VMSs will not be restricted to dense clusters, since a collision runaway is no longer necessary for achieving masses $\gtrsim 150 M_\odot$ (see Table 1).

3 ECLIPSE PROBABILITY

The fraction of massive O-type stars in binaries f_b is observed to be extremely high $> 70\%$ (Chini et al. 2012), and approaches 100% in some environments (Mason et al. 2009; Bosch, Terlevich & Terlevich 2009). Although there is no related observational data on VMSs, numerical simulations indicate that the collision runaway product in young, dense star clusters is generally accompanied by a companion star (Portegies Zwart, private communication).

As the period distribution for our hypothetical VMS binaries is unknown, we assume their periods share the same cumulative distribution function (CDF) as the periods of massive binaries determined from observations. The CDF of the orbital period (p , in days) for massive binaries follows a ‘broken’ Öpik law, i.e. a bi-uniform distribution in $\log p$, with the break at $p = 10$ (Sana & Evans 2011). There is an overabundance of short period binaries, with 50% to 60% of binaries having periods less than 10 days. The corresponding probability distribution function $PDF(p)$ of the orbital period is:

$$PDF(p) = \frac{1}{\ln 10} \times \begin{cases} \frac{5}{7p}, & \text{for } 10^{0.3} \leq p \leq 10 \\ \frac{1}{5p}, & \text{for } 10 < p \leq 10^{3.5}, \end{cases}$$

with the normalization $\int PDF(p) dp = 1$.

By integrating over uniformly distributed inclinations, it is easy to show that the eclipsing probability of a binary system at any depth is $P_e(a) = \frac{R_t}{a}$, where $R_t = R_1 + R_2$ is the sum of the radii of both components in the binary, and a is the orbital distance. From Kepler’s third law, we can express the eclipsing probability as a function of p instead:

$$P_e(p) = R_t \left(\frac{2\pi}{p}\right)^{\frac{2}{3}} (GM_t)^{-\frac{1}{3}}, \quad (4)$$

where $M_t = M_1 + M_2$ is the total system mass. Therefore, integrating over the period distribution, the probability that a massive binary will be an eclipsing binary to an observer on Earth is

$$\begin{aligned} P_e &= \int P_e(p) PDF(p) dp \\ &\approx 0.053 \left[\frac{R_t}{R_\odot}\right] \left[\frac{M_t}{M_\odot}\right]^{-\frac{1}{3}}. \end{aligned} \quad (5)$$

For convenience, we ignore any effects of eccentricity; tidal evolution will rapidly circularize the orbit for binaries with periods below $p = 10$ days, which account for 88% of the above eclipsing systems. Dynamical effects would harden a wide-separation massive binary system in the core of a dense cluster on a timescale much shorter than 1 Myr. Since three-body interactions tend to eject the lightest star, the companion to the VMS will likely be a massive star, though not as massive as the runaway product.

The large radii of VMSs coupled with their high binary fraction (and short period binaries being common), imply significant eclipsing probabilities for VMSs. Using R136a1 as an example of the primary star, with a radius $\sim 35 R_\odot$, and a secondary Sun-like star, the eclipsing probability is 29%, while for a more massive secondary star more common in the core of a young massive star cluster, e.g. a B0 star of mass $\sim 18 M_\odot$ and radius $\sim 7 R_\odot$, the eclipse probability is 34%. Note that the eclipsing binary probability in equation (5) is not sensitive to the secondary star parameters, as long as its radius is small relative to the primary.

Assuming a companion B0 star, we list the eclipsing binary probabilities for our candidate VMSs in Table 1, calculated from equation (5), except that we limit the integration over p to periods corresponding to orbital distances exceeding both the radius of the VMS and the Roche limit for the companion. This restriction reduces P_e , and leads to the larger VMSs having slightly smaller eclipsing proba-

Table 1. Possible very massive stars in star clusters and their eclipse probabilities. The predicted runaway collision product mass m_r is calculated from equation (1). Another possible VMS stellar mass m_u is calculated via the relationship between the cluster mass and its most massive star in equation (3). All masses are in units of M_\odot , the cluster age is measured in Myr, and the virial radius r_{vir} is in units of pc. If we optimistically choose the largest mass of m_r and m_u for the primary mass M_1 , we can calculate its luminosity L_1 (in L_\odot) and radius R_1 (in R_\odot) using the models of Bromm, Kudritzki & Loeb (2001), assuming a characteristic stellar metallicity (Z/Z_\odot) = 0.3. We calculate the eclipsing probability P_e assuming that the companion is a B0 star, although the result is weakly sensitive to the companion mass. For generality, the expected transit depth $\langle\delta\rangle$ is averaged over a uniform distribution in the binary mass ratio q , up to a companion mass of $10^2 M_\odot$, assuming non-grazing orbits, i.e. $\delta \approx (R_2/R_1)^2$. For all VMS candidates below, the expected dip in luminosity from the eclipse is $\sim 10^6 L_\odot$.

Galaxy	Name	Ref	Age	$\log M_C$	r_{vir}	m_r	m_u	L_1	R_1	P_e	$\langle\delta\rangle$
Milky Way	Arches	1	2.0	4.30	0.68	192	103	5e6	44	39%	16%
LMC	R136	2,3,4	3.0	4.78	2.89	406	170	1e7	61	36%	8%
SMC	NGC 346	5	3.0	5.60	15.28	640	397	2e7	76	34%	5%
M33	NGC 604	6	3.5	5.00	48.21	97	213	6e6	46	38%	15%
NGC 1569	C	6	3.0	5.16	4.50	672	252	2e7	77	34%	5%
NGC 4214	I-A	6	3.5	5.44	28.69	305	337	1e7	56	36%	10%
NGC 4214	I-B	6	3.5	5.40	9.85	619	323	2e7	74	34%	6%
NGC 4214	II-C	6	2.0	4.86	23.43	129	185	5e6	43	39%	17%
NGC 4449	N-2	6	3.0	5.00	3.57	565	213	2e7	71	35%	6%
NGC 5253	IV	6	3.5	4.72	5.26	271	160	8e6	51	37%	12%

(1) Figer, McLean & Morris (1999); (2) Hunter et al. (1995); (3) Mackey & Gilmore (2003); (4) Andersen et al. (2009); (5) Sabbi et al. (2008); (6) Maíz-Apellániz (2001).

bilities; nevertheless, the eclipsing probabilities for all VMS candidates exceed 1/3.

4 OBSERVABILITY OF TRANSIT

VMSs have spectacular luminosities in the range of $10^7 L_\odot$; for example, R136a1 is observed to have $\sim 8.7 \times 10^6 L_\odot$. Even at a distance of 3 Mpc – roughly the distance of the farthest host galaxy in Table 1 – a star like R136a1 would still have an apparent bolometric magnitude of 14.8. However, VMSs with $T_{eff} \sim 5 \times 10^4$ K emit primarily in the ultraviolet, requiring bolometric corrections of $BC \sim 4.6$. Still, such a VMS will be within the V-band limiting magnitude of ground-based 1-meter telescopes. For the VMS candidates in Table 1, with a hypothetical B0-star companion, the transit depth exceeds $10^5 L_\odot$ in all cases, which at 3 Mpc is just within the single-visit limiting magnitudes of future synoptic surveys such as Pan-STARRS¹ and the Large Synoptic Survey Telescope². Of course, given the shortlist of host clusters in Table 1, one can use deep, targeted observations of the individual clusters with existing telescopes, instead of uniform field surveys.

However, in massive binaries, the mass ratio between the primary and secondary star $q = M_2/M_1$ is observed to have a flat distribution (Sana & Evans 2011). Unlike the transit probability, the transit depth is very sensitive to the companion star radii, so using a B0 star as the companion may be overly conservative. Since only one VMS is expected to form in the collision runaway scenario, here we assume the distribution of companion star masses is uniform between 1 to $100 M_\odot$. Using typical mass-radius relationships, we show in Table 1 the expected transit depth $\langle\delta\rangle$ integrated over the range of companion star radii. Figure 1 illustrates

sample light curves for a VMS binary at 3 Mpc with different companion star masses and radii at different inclinations.

For clusters outside the Milky Way and its satellites, it is currently impossible to resolve a VMS from other massive stars in a dense cluster core. Hence, we consider the luminosity of the host cluster as a contaminating third light source to the eclipsing binary light curve. If the VMS is present, it will contribute a significant fraction of the bolometric luminosity of the cluster (at least 10% and exceeding 50% in some cases), and an even larger fraction of the UV flux. The integration time t needed to reach a target signal-to-noise ratio SNR for detecting a transit can be approximated as:

$$\begin{aligned}
 t \approx & 6 \text{ seconds} \times \left[\frac{L_C}{10^8 L_\odot} \right]^{-1} \left[\frac{d}{3 \text{ Mpc}} \right]^2 \\
 & \times \left[\frac{f_{band}}{0.2} \right]^{-1} \left[\frac{E_{band}}{10 \text{ eV}} \right] \left[\frac{A}{4 \times 10^4 \text{ cm}^2} \right]^{-1} \\
 & \times \left[\frac{SNR}{10} \right]^2 \left[\frac{f_{VMS}}{0.1} \right]^{-2} \left[\frac{\delta}{10\%} \right]^{-2}
 \end{aligned} \tag{6}$$

where L_C is the bolometric luminosity of the cluster, d is the distance to the cluster, f_{band} is the fraction of total flux that is observed (due to the spectral energy distribution, filter bandpass, CCD response, atmospheric transmission etc.), E_{band} is the characteristic observed photon energy, A is the collecting area of the telescope, f_{VMS} is the fraction of total observed flux from the VMS primary, and again δ is the transit depth.

Note that the Hubble Space Telescope (HST) would collect $\gtrsim 10^4$ UV photons per second from a $10^7 L_\odot$ VMS even at a distance of 3 Mpc, thus detecting a $\delta \sim 10\%$ transit depth at $SNR = 10$ in tens of seconds of integration time. Obscuration by dust along the line-of-sight may reduce the observed UV flux. For V-band observations, a very young $\sim 10^5 M_\odot$ cluster can be as bright as $M_V \approx -12$, while a $300 M_\odot$ VMS will have $M_V \approx -8$, i.e. the VMS will only contribute $f_{VMS} \sim 2.5\%$ of the cluster light in the visible band. Nevertheless, a 2-meter ground-based telescope will

¹ <http://pan-starrs.ifa.hawaii.edu/public/>

² <http://www.lsst.org/lsst/>

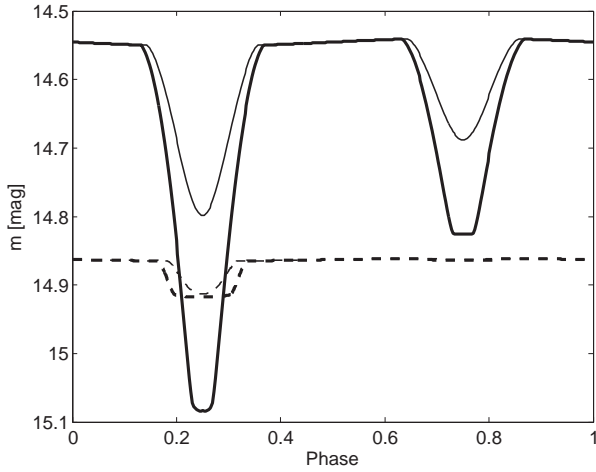


Figure 1. Example light curves for a VMS eclipsing binary. The primary has parameters similar to R136a1, while the secondary is either a $18M_{\odot}$ (dashed line) or $100M_{\odot}$ (straight line) star, with appropriate radii and luminosities. The apparent magnitude m (bolometric) is plotted for these systems at 3 Mpc. The thick and thin lines correspond to inclinations of 90° and 70° , respectively; the period is 5 days in both cases. Reflections and limb-darkening using the model of Diaz-Cordoves, Claret & Gimenez (1995) are taken into account, but ellipsoidal variation is ignored.

need less than an hour of integration time to detect the transit, which is eminently feasible as the transit duration $\tau \sim p(R_1/\pi a) \propto p^{1/3}$ for a VMS eclipsing binary will be > 10 hours for all relevant orbital periods.

Other less massive eclipsing binaries in the host cluster will also contaminate the light curve, but their transit depths will likely be negligible compared to the VMS's luminosity. Non-binary random occultations of the central VMS can replicate a large transit depth, but using a King model for the cluster density profile (King 1966), we find these events occur less than once every 10^6 years.

5 STELLAR MASS DETERMINATION

The extraordinary luminosity of a VMS should allow its radial velocity to be measured. However, the mass ratio q , critical for model-independent determination of the individual masses, can only be found when the radial velocities are determined for *both* components of the binary. Such double-lined spectroscopic binaries are easily observable when the components have similar luminosities, within a factor of 5 of each other (Kallrath & Milone 2009). As the luminosity of massive stars near the Eddington limit scales with mass, this criteria roughly corresponds to $q > 0.2$, which for an uniform distribution in $q \in (0, 1)$ is quite likely to occur.

Nevertheless, if the companion is small, and only spectral lines from the VMS are detected, then the mass ratio q cannot be unambiguously obtained. Instead, the mass of the VMS can be expressed as a single function of q :

$$M_1 = \frac{(1+q)^2}{q^3} \frac{1}{\sin^3 i} f(M_1, M_2, i), \quad (7)$$

where $f(M_1, M_2, i)$ is the *mass function*, which can be calculated using quantities derivable from the spectroscopy of

a single-lined spectroscopic binary, and the inclination i is derivable from the eclipsing binary light curve. Unfortunately, equation (7) varies sharply as $\propto q^{-3}$ for $q \ll 1$. Since q can be as small as ~ 0.01 for VMSs in Table 1, crude constraints on the mass ratio, e.g. $q < 0.2$ (when light from the secondary is not observed) cannot establish tight minimum stellar mass constraints on the VMS primary.

However, since a total eclipse $\delta \rightarrow 100\%$ is extremely unlikely given the large radii of VMSs, if the mean value $\sim 10^6 L_{\odot}$ dip in the light curve is in fact observed, it will immediately imply the existence of a star $\gtrsim 10^2 M_{\odot}$. Hence, although sophisticated light-curve fitting with stellar models would be required, eclipsing single-lined spectroscopic binaries still offer an attractive avenue for inferring the presence of a VMS greatly exceeding the $150 M_{\odot}$ stellar mass limit.

6 DISCUSSION

A search for periodic flux variations (as shown in Fig. 1) due to transits of the VMS candidates in Table 1 would be of considerable interest. Although Crowther et al. (2010) made robust arguments against R136a1 being a wide separation binary or an equal-mass binary, this source could still involve a short-period, unequal-mass binary system. The Arches cluster is observed to have no stars currently above the $150M_{\odot}$ mass limit, but Crowther et al. (2010) also found with contemporary stellar and photometric results that the most luminous stars in the Arches cluster had initial masses approaching $200M_{\odot}$.

The radii of VMSs are dependent on their metallicities and rotation (Langer et al. 2007). If the VMS radii in Table 1 were smaller by $\sim 25\%$ (e.g. at much lower metallicities), all listed eclipsing probabilities would still remain above $1/3$, but the expected transit depth would increase up to $\langle \delta \rangle \sim 20\%$. As for the companion star, for most O stars, the point of unity Thomson optical depth occurs close to the hydrostatic radius, but when stellar mass loss exceeds $\sim 10^{-5} M_{\odot} \text{ yr}^{-1}$, the photosphere $\tau \sim 1$ occurs in the wind itself, effectively increasing the star's radius. This occurs for Wolf-Rayet companions (Lamontagne et al. 1996) and for companions $\gtrsim 60M_{\odot}$ (Vink, de Koter & Lamers 2000), in which case our eclipse probabilities and transit depths are too conservative.

Binaries can be broadly classified into detached systems, where neither component fills its Roche lobe, versus semi-detached or over-contact systems, where at least one component exceeds its Roche lobe. VMSs in detached binaries have much more sharply defined eclipses, and more importantly, they do not undergo mass transfer and lose mass to their companion. To find the probability that our VMS candidates in Table 1 are detached eclipsing binaries, we limit the integration in equation (5) to periods $p \gtrsim 5$ days, corresponding to orbital distances where the Roche lobe of the VMS is always greater than its radius (Eggleton 1983). For our VMS candidates, the detached eclipsing binary probability is $\approx 17\%$, i.e. roughly half of all eclipsing systems.

However, non-pristine massive stars can also lose mass via strong winds driven by radiation pressure, with a mass loss rate increasing with metallicity. Post main-sequence VMSs can also lose mass eruptively or via pulsational instabilities, although mass loss near the end of the star's life

(e.g. the pulsational pair-instability) is not likely to change the observability of our VMS candidates. Under extraordinary mass loss via winds, Glebbeek et al. (2009) found the highest mass attained by a collision runaway product to be $\sim 400M_{\odot}$, although the star remained at this mass range for only ~ 0.2 Myr. On the contrary, Suzuki et al. (2007) found that stellar mass loss does not inhibit the formation of a VMS of $\sim 10^3M_{\odot}$.

If VMSs do in fact form via collision runaways in young, dense star clusters, and retain sufficient masses at the end of their lives, they may explode as pair-instability supernovae (PISNe) (Yungelson et al. 2008). The creation rate of runaway products is in fact consistent with the current observed PISN rate (Pan, Loeb & Kasen 2012). However, the most massive VMSs may collapse directly into an intermediate mass black hole (IMBH) via the photodisintegration instability (Woosley, Heger & Weaver 2002). Tentative evidence has been claimed for IMBHs at the center of old globular clusters (Lou & Wu 2012), and extragalactic ultra-luminous x-rays sources associated with young star clusters (Ebisuzaki et al. 2001; Farrell et al. 2009). The identification of VMSs that can serve as the progenitors of PISNe and IMBHs will help move these extreme astrophysical objects from the realm of speculation into reality.

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³ <http://wise-obs.tau.ac.il/~eran/matlab.html>